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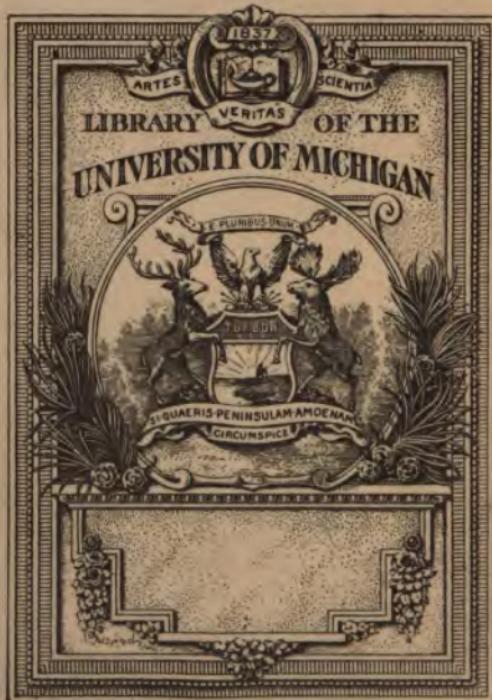
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**NEW METHOD
IN
MULTIPLICATION
AND
DIVISION**



WILLIAM TIMOTHY CALL



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A NEW METHOD IN
MULTIPLICATION
AND DIVISION

NEW METHOD
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AND
DIVISION

By
WILLIAM TIMOTHY CALL
=

Price, 50 Cents

HAWTHORNE, N. J.
C. M. POTTERDON
1913

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WILLIAM TIMOTHY CALL

PREFACE

The method here shown is the outcome of my efforts to simplify the process of multiplying and dividing numbers. The chief purpose in this endeavor was to indulge my own curiosity as to the possibility of doing such a thing.

Because it was a pleasing diversion for an idle hour, when relaxation was desirable, the quest was resumed as the mood for it came along.

In that way, while speculating on the notion of how to factor the number 7, I wrote $(3 \times 2) + 1$, then $(5 \times 1) + 2$, then $(10 \times 1) - 3$, and so on as fancy led. While rambling along in this fashion I noticed that 5 is a kind of pivotal number to the digits.

From this observation I passed on to experiments out of which came the method given in the succeeding pages. It is not a matter of reciprocals, or of complements,

or of factoring, but may be called a method of conversion. It is not a matter of importance. It is a curiosity. In Part V another way of finding the product of two numbers is given.

W. T. CALL.

NEW YORK,

March, 1913.

A NEW METHOD IN MULTIPLICATION AND DIVISION

PART I NUMBER

Once upon a time,—but first let me ask:
What is number?

I judge from your puzzled look that you
know, but “can’t exactly tell.”

Well, many persons of seemingly more
powerful intellect than most of us would
care to have are in the same fix. One of
them excused himself by saying this:

“There is no more difficult mathematical
problem than to define what number really
is.”

Another said that he didn’t just reckon
number is a unit or a collection of units, but

a continuous thing, like time or space. What he meant may be something like this:

It is night. You see in the distance a string of electric lights stretching across the street—that is, you see the lights, but not the string. The lights happen to mark certain points on the string, but you regard the string itself as continuous.

Years ago, down in Bangor, Maine, there lived a happy half-witted fellow by the name of Dan Thompson. They used to say that Dan did not have all his buttons, but it was admitted that he had found a question (from Christy's Minstrels, perhaps) that no one in that bright little city was ever able to answer.

It was:

“Why is a hen?”

As a child I surrendered to happy Dan's question, and I now renounce, in the same way and spirit, all hope of ever being able to find out what number really is.

No doubt the first practical use of number is in counting. That seems to be the lowest foundation course of bricks in the whole mathematical structure. Addition, subtraction, multiplication, and division are merely artifices for rapid counting. Proof:

If you play poker or bridge, and in a resulting nightmare you see seven pawnbrokers' signs in a row, you do not have to count each individual ball to know there are twenty-one in all. Nor do you have to add one golden group of three to another. In your dream you would naturally and deservedly multiply your distress by observing that 7 times 3 are 21.

The ancients had no arithmetic. They could not multiply one number by another as we do now. They used the abacus in their calculations, just as a Chinese laundryman does to-day. You have often seen that primeval calculating machine—that imperishable toy composed of beads strung on wires. Not until modern times did multiplication and all the other mathematical processes as we know them become possible. So we are told, but—see Part V.

It is an historical fact that the ancients didn't know nothing. The moderns got nothing, and the whole of arithmetic came from nothing. We could not get along without nothing now.

Perhaps you will understand this point better from the following quotation:



"The introduction of the symbol 0 is the greatest step ever taken in the history of arithmetical science, and one that completely escaped the Greeks and Romans."

The zero (or cipher, or naught) was surely a happy thought. It passed from the Hindus of India to the Arabs of Arabia, Egypt, and Coney Island. The figures we now use came in the same package from the same people. These figures took the place after awhile of the letters used by the ancients, I, V, X, L, for example. Our present figures, however, are deformed descendants of letters.

It was the zero (which means empty space) that brought about in some way the idea that a figure represents ten times as much in any place as in the succeeding place. That is, the zero gave birth to the position system in number.

For illustration: Put down four zeros, thus: 0000. Now put the figure 3, for instance, in place of each of these empty spaces, and you have this: 3333. That shows that the figure 3 represents 3 thousand, or 3 hundreds, or 3 tens, or 3 units, according to the *place* it occupies.

[REDACTED]

The old Hindu who first thought of this scale of ten (units, tens, hundreds, thousands, tens of thousands, etc.) got his notion probably from Nature's own counting machine—the ten fingers of man.

Perhaps if Adam had had twelve fingers we would now make our reckonings on a scale of twelve. In that case 3333 would stand for 3 greatgross, 3 gross, 3 twelves, 3 units. In other words we would become accustomed to think in dozens (as some tradesmen do) instead of in tens. Twelve is a better basis than ten, because it can be divided by 2, 3, 4, and 6, without a fraction, whereas ten takes only 2 and 5.

That is enough to give a hint of what a queer thing number really is. But there is a little more deadwood I would like to get out of the way before coming to the business of multiplication and division.

The science of number and the science of form are—mathematics. The science of number is covered by arithmetic and its big children, algebra and calculus. The science of form (triangles, circles, cubes, and such) is covered by that tough citizen, geometry, and all its horrid offspring.

[REDACTED]

It would seem that the mathematics ~~is~~ composed of arithmetic, algebra, geometry and calculus, with their legitimate progeny would be enough to satisfy us all. But we are compelled to recognize the fact that human beings there is an eternal yearning to get more than enough. We observe the effects of this craving wherever we look. Give a man a million dollars, and he will work for more. Give a woman an onion, and she will cry for more.

So we find in mathematics such things as hyper geometry, and various other hypotheses and vipers. Take the fourth dimension, for instance. By the way would you like to know what the fourth dimension is, just for fun?

I don't know what it is, but I will tell you so you may know. You can comprehend it if you will take an ordinary orange and turn it inside out without breaking the skin.

It is admitted by our most fashionable thinkers that the good old-time dimensions, length, breadth, and thickness, are all the dimensions anything can possibly have. Some things, a square, for instance, have only two dimensions, length and breadth. A line has

only one dimension, length. A cube, however, has all three dimensions, length, breadth and thickness. Working forward from a two-inch line, for example, to a square, then to a cube, this is the way to represent them mathematically:

$2 =$ a line
 $2^2 =$ a square
 $2^3 =$ a cube
 $2^4 =$ what?

That "what" is the fourth dimension, and there is nothing to keep you from going on to other dimensions, if you want to. Theoretically there ought to be a fourth dimension, and if you love a pure idea, as our old friend Plato did, you will find in the fourth dimension a wonderful plaything, the mere possession of which will mark you as a person of exalted intellect.

We will now come back to our dirt. The A B C of arithmetic, algebra, and calculus, is something like this:

Say the side of a square is two inches, then the square itself is $2 \times 2 = 4$ inches. So we express this fact in arithmetic thus:

$$4 = 2^2$$

In algebra we may use the letters a and b instead of the figures 4 and 2, thus:

$$a = b^2$$

In arithmetic, you see, we are dealing with a specific square and no other. In algebra we are dealing with a square of any size; that is, with all squares.

Now in calculus they say the size of the square depends on the length of its side. That is, the square increases or diminishes as the side changes in length. You may say that this is an absurdly simple principle. So it is, but it is a very powerful idea in mathematics, just as that other simple thing, gravity, is powerful in physics. So we will now take x and y instead of a and b or 4 and 2, and say the size of x depends on the size of y , or in technical language, x is a "function" of y . The way that notion is expressed in calculus is :

$$x = f(y)$$

You need not bother your head about why this artifice is more powerful in problems than arithmetic or algebra, because it is not used except in complex questions, such as finding out how things change, or rather at what rate they change. Thus the velocity of

a cannon ball in the air changes from instant to instant. It is not a fixed quantity, like the side of a known square.

Thus, roughly speaking, in arithmetic we have known quantities, in algebra unknown quantities, in calculus changing quantities.

Do you like Petrarch?

I do not mean something to eat. I mean Petrarch, who wrote books. You must not mix him up with Plutarch, who also wrote books, but centuries before Petrarch. Petrarch wrote the best sellers of his time, and if Columbus could read them he probably had one when he came to America; for it was in 1471 that a Petrarch reprint had the Hindu, or Arabic, figures, as we call them, at the top as page numbers. That was the first time figures were used for this purpose, and that practice has been followed by all the best sellers to this day.

Columbus came here, I think, in 1492. Well, in 1478 the first arithmetic came out. Four years later another arithmetic appeared. These pioneer books explained the use of the zero system. They caused the shape of the figures to be accepted as fixtures, and thus put a stop to the changes in their shape that had been

going on. The old figures in use in the cloisters of the middle ages, if seen in a newspaper of to-day, would be taken for an advertisement of hooks and eyes, safety pins, and bangles.

So you see arithmetic is a toddler in age and needs all the help it can get, however ungrateful it may be.

PART II.

MULTIPLICATION

The old way:

$$\begin{array}{r} 83904 \\ 62517 \\ \hline 587328 \\ 83904 \\ 419520 \\ 167808 \\ 503424 \\ \hline 5245426368 \text{ } Ans. \end{array}$$

The new way:

$$\begin{array}{r} 83904 \\ 62517 \\ \hline 419520 \\ 83904 \\ 167808 \\ 419520 \\ 83904 \\ 419520 \\ 167808 \\ \hline 5245426368 \text{ } Ans. \end{array}$$

Definitions:

The number at the top is the multiplicand; the number under it, the multiplier; the numbers between the lines, the partial products; the number at the bottom, the product, or answer.

Difference in favor of the old way:

Five partial products in the old way, against seven partial products in the new way.

Difference in favor of the new way:

The partial products in the new way are copied from a Key. The work is mechanical. If there is an error in the partial products, it can be detected by inspection.

In the old way the partial products are obtained by multiplying each of the five figures of the multiplicand by each of the five figures of the multiplier, making in this example twenty-five mental struggles. They are complex operations—multiplying, putting down, carrying. If there is an error in the partial products, it can not be detected by mere inspection, but must be sought by doing the work all over again.

In the old way the partial products are slowly put down backward—from right to

left. In the new way the partial products are put down like words—from left to right—and, again like words, may be dictated as rapidly as a stenographer can put them down.

The operation of adding the partial products is the same in the new way as in the old way.

Note: A written explanation of any mathematical process, however simple that process may be, needs more than a single reading. Under the heading "Analysis," near the close of the following explanation, the reader will see the complete operation (unabridged), which shows just what takes place.

The Key:

The first thing the learner should do is to write the Key on a separate piece of paper. The Key is always 1, 2, and 5 times the multiplicand. A different multiplicand, of course, makes a different Key. The partial products in all cases are the numbers of the Key repeated, and no others. It is apparent that it is not actually necessary to make a separate Key, but it is also apparent that a Key on a separate piece of paper is convenient, positive, and leaves the mind free.

The way to put down the Key for the ex-

ample is here shown; 1 standing for once,
2 for twice, 5 for five times the multiplicand:

KEY

$1 = 8\ 3\ 9\ 0\ 4$
$2 = 1\ 6\ 7\ 8\ 0\ 8$
$5 = 4\ 1\ 9\ 5\ 2\ 0$

Refer to the example, and observe that the partial products are these numbers repeated, and no others.

The ease and certainty with which the Key is made are seen in the fact that it is nothing more than the multiplicand itself, two times the multiplicand and one-half the multiplicand with a cipher added (since multiplying by 5 is the same as multiplying by 10 and dividing by 2).

The Scale:

It has been said that a separate Key is desirable, but not essential. It has been shown that a different multiplicand makes a different Key.

The Scale, on the other hand, is so obvious that it is not necessary to write it down at all. Besides, it is always this:

SCALE

1	=	1
2	=	2
3	=	5 — 2
4	=	5 — 1
5	=	5
6	=	5 + 1
7	=	5 + 2
8	=	10 — 2
9	=	10 — 1

The first column of the Scale shows the digits. The second column shows their equivalents. That is, the digits are converted into the Key figures, 1, 2, 5. Illustration: The Scale shows that $7 = 5 + 2$. To multiply a number by 7 you multiply it by 5 and also by 2, because the Key shows what 5 times the number is and what 2 times the number is.

Putting down the Partial Products:

The partial products are copied from the Key, and are written from left to right, like words.

It is important to remember that each partial product begins directly under the figure of the multiplier that produced it; thus:

$$\begin{array}{r} 50 \\ 23 \\ \hline 100 \\ 150 \\ \hline 1150 \end{array} \text{Ans.}$$

Also:

$$\begin{array}{r} 50 \\ 21 \\ \hline 100 \\ 050 \\ \hline 1050 \end{array} \text{Ans.}$$

In the second case the last partial product, 050, begins with zero—nothing—an empty place, because 1×5 makes less than 10. Hence the rule:

If the Scale figure times the first figure of the multiplicand makes less than 10, start that partial product with zero.

Thus:

$$\begin{array}{r} 40 \\ 12 \\ \hline 040 \\ 080 \\ \hline 480 \end{array} \text{Ans.}$$

In this case both partial products begin with zero because 1×4 makes less than 10, and 2×4 makes less than 10.

Also :

$$\begin{array}{r} 50 \\ 18 \\ \hline 050 \\ 500 \\ -100 \\ \hline 900 \end{array} \text{Ans.}$$

In this case 1×5 makes less than 10, thus starting the first partial product with zero. The other partial products do not start with zero, because the Scale figures for 8 are 10 — 2, and neither 10×5 nor 2×5 makes less than 10.

The Minus Sign :

A minus Scale figure makes a minus partial product. The minus sign should be inserted before each figure of a minus partial product as a guide to the eye.

At first the minus figures in a column of partial products should be added separately, and their sum deducted separately from that of the other partial products. Thus :

$$\begin{array}{r}
 987 \\
 109 \\
 \hline
 0987 \\
 9870 \\
 -0987 \\
 \hline
 108570 \\
 987 \\
 \hline
 107583 \text{ Ans.}
 \end{array}$$

KEY

1 = 987
2 = 1974
5 = 4935

When familiar with the new method, the minus figures may be deducted directly as each column of the partial products is added, and the true product put down without a second operation.

ADDITIONAL EXAMPLES:

$$\begin{array}{r}
 17 \\
 191 \\
 \hline
 017 \\
 170 \\
 -017 \\
 \hline
 017 \\
 \hline
 3417 \\
 170 \\
 \hline
 3247 \text{ Ans.}
 \end{array}$$

KEY

1 = 17
2 = 34
5 = 85

$$\begin{array}{r}
 989 \\
 94 \\
 \hline
 9890 \\
 -0989 \\
 \hline
 4945 \\
 -0989 \\
 \hline
 103845 \\
 10879 \\
 \hline
 92966
 \end{array}
 \quad
 \begin{array}{l}
 \textit{Ans.}
 \end{array}$$

KEY

1 = 989
 2 = 1978
 5 = 4945

This example will serve as a good illustration of how the minus figures in a column may be directly united with the plus figures. The sum of $-9 + 5$ (the first right hand column) is -4 , which equals 6, because $10 - 4 = 6$. The sum of the next column is -1 (the 10 that was taken from it by the first column) $-8 + 4 - 9 = -14$, which equals 6, because $20 - 14 = 6$. The sum of the third column is -2 (the 20 that was taken) $-9 + 9 - 8 + 9 = -1$, which equals 9, because $10 - 1 = 9$. The sum of the fourth column is -1 (the 10 taken) $+4 - 9 + 8 = 2$. Then $9 = 9$. Uniting minus and plus figures is simpler in practice than this example makes it appear.

If the learner finds the minus signs confusing, the minus partial products may be

separated from the others. The multiplier in this example being 94, first place is directly under 9, second place directly under 4. The multiplier is here put down twice in order to clearly show the place in which each of the minus partial products starts:

$$\begin{array}{r} 989 \\ 94 \\ \hline 9890 \\ 4945 \\ \hline 103845 \\ 10879 \\ \hline 92966 \end{array} \quad \begin{array}{r} 94 \\ -0989 \\ -0989 \\ \hline -10879 \end{array}$$

Ans.

Note in the above example that the total minus product (10879) begins under the 9 of 94.

$$\begin{array}{r} 1002 \\ 99 \\ \hline 10020 \\ -01-0-0-2 \\ 10020 \\ -01-0-0-2 \\ \hline 99198 \end{array}$$

Ans.

KEY

1 = 1002
2 = 2004
5 = 5010

$$\begin{array}{r}
 478 \\
 654 \\
 \hline
 2390 \\
 0478 \\
 2390 \\
 2390 \\
 \hline
 -0478 \\
 \hline
 312612 \text{ Ans.}
 \end{array}$$

KEY

1 = 478
2 = 956
5 = 2390

$$\begin{array}{r}
 78659 \\
 9756 \\
 \hline
 786590 \\
 -78659 \\
 393295 \\
 157318 \\
 393295 \\
 393295 \\
 \hline
 78659 \\
 \hline
 767397204 \text{ Ans.}
 \end{array}$$

KEY

1 = 78659
2 = 157318
5 = 393295

Analysis:

To demonstrate the correctness of the new method in multiplication, the entire make-up of the process may be shown as follows:

$$\begin{array}{r}
 3273 \\
 493 \\
 \hline
 \end{array}$$

KEY

1 = 3273
2 = 6546
5 = 16365

Multiplier completely converted: $493 = (500 - 100) + (100 - 10) + (5 - 2)$.

Operation:

$$\begin{array}{rcl} 500 & = & 1636500 \\ 100 & = & 327300 \\ 5 & = & \underline{16365} \\ & & 1980165 \\ & & 366576 \\ \hline & & 1613589 \text{ } Ans. \end{array} \quad \begin{array}{rcl} -100 & = & -327300 \\ -10 & = & -32730 \\ -2 & = & \underline{-6546} \\ & & -366576 \end{array}$$

Conclusion:

While it may be well for the beginner to write out the complete process, the practitioner will dispense with all unnecessary details.

The Scale is obvious and the Key is itself in the partial products. With a separate Key, however, the partial products may be dictated like words, thus:

654 \times 798

KEY

1 = 798
2 = 1596
5 = 3990

Dictate:

"First place 3990, first place 0798, second place 3990, third place 3990, third place minus 0798."

The partial products as taken down for the addition then appear thus:

$$\begin{array}{r} 3990 \\ 0798 \\ 3990 \\ 3990 \\ -0798 \\ \hline 521892 \text{ Ans.} \end{array}$$

PART III

DIVISION

Division is continuous subtraction. To divide 12 by 3 is to subtract 3 from 12 four times. The dividing number, 3, is the divisor; the number to be divided, 12, is the dividend; the result is the quotient, or answer.

To divide one large number by another large number in the old way a figure is placed in the quotient on trial. If the divisor multiplied by that figure makes too much or too little, another figure for the quotient is tried.

In the new way the quotient figure and the number to be subtracted from the dividend are written like words, and are obtained from a Key.

The old way:

12) 1824 (152 *Ans.*

$$\begin{array}{r}
 12 \\
 \underline{\times} \\
 62 \\
 \hline
 60 \\
 \hline
 24 \\
 24
 \end{array}$$

The new way:

$$\begin{array}{r}
 1824 \left(\begin{array}{r} 150 \\ 1800 \\ \hline 24 \end{array} \right. \\
 \hline
 24 \quad 152 \quad \text{Ans.} \\
 \hline
 24
 \end{array}$$

KEY

1	=	12
2	=	24
4	=	48
8	=	96
<hr/>		
15	=	180

The Key is always made in the way here shown, namely: 1 = the divisor, 2 = twice the divisor, 4 = twice that, 8 = twice that, 15 = the sum. As the Key always starts with the divisor, it is not necessary to write the divisor in front of the dividend or to pay any attention to it other than as part of the Key.

What was done in the new way is this:

The Key shows that 180 is the convenient number to subtract from the dividend, 1824. The Key shows that $15 = 180$; consequently 15 is the number to write in the quotient. One cipher was joined to 180; consequently one cipher must be joined to 15. The new dividend is 24. The Key shows that $2 = 24$; consequently 2 must be put in the quotient.

That is the complete process in all cases. The Key is composed of five parts because that is the most convenient kind of a Key to have.

EXAMPLES:

62517) 5245426368 (80000

5001360000	2000	
244066368	1500	KEY
125034000	400	1 = 62517
	4	2 = 125034
119032368	83904	4 = 250068
93775500		8 = 500136
25256868		15 = 937755
25006800		
250068		

Ans.

83904)	5245426368	(40000	
	3356160000	20000	
	1889266368	2000	
	1678080000	400	
		100	
	211186368	15	KEY
	167808000	2	1 = 83904
	43378368	62517	2 = 167808
	33561600		4 = 335616
			8 = 671232
	9816768		
	8390400		15 = 1258560
	1426368		
	1258560		
	167808		

In practice the operation may be abbreviated, as in ordinary division, thus:

432) 354240	(820	<i>Ans.</i>		KEY
	3456			1 = 432
	86			2 = 864
				4 = 1728
				8 = 3456
				15 = 6480

PART IV

PRODIGIES

Several years ago I went to the twenty-fifth anniversary dinner of my college class. We ate and drank and talked and smoked. Baseball reminiscences came out. I used to play left field on the Bowdoin nine. The conversation became personal and rapid, with many "Do-you-remembers." Suddenly a sedate lawyer broke his silence with:

"Oh, Billy, I shall never forget it—that catch—that catch!"

I felt like a prodigy.

Smoking up at double speed, I kept down the question I longed to vent: "Which one of the *many*?" I had an inchoate trepidation that he might reply: "The one you *made*."

On my way back to the big city by the sea

I tried to recall that catch. There was a low fence in left field that was sometimes reached by a big hit. One of the heavy batters of the Bates College nine lifted the ball in a long graceful parabola for that destination. Running at full speed I leaped the fence and caught the ball *en passant*, as they say in chess.

I had always been under the impression that it was my predecessor in left field, Blyndie Fuller, who made that catch; but to err is human, and I now have no doubt that I made it.

Another specimen: I was in a subway car in New York that for some reason unknown to me was not crowded. Leaning comfortably on my elbow to gaze through the window at the scenery in the near distance, I heard a peculiarly clear voice back of me. It was the voice of a refined gentleman, who was explaining to a girl the funny pictures in one of New York's most famous monstrosities—a daily newspaper for the feeble-minded. That is, that is the way I regarded the kind of stuff that paper featured until I heard this gentleman's astounding interpretation of the symbolism of those wonderful

pictures and the meaning of the badly printed lines of text worked in here and there. I listened spellbound for a few minutes, and then turned and gazed in abject wonder at the gentleman, who was about five years old, while the girl was about thirty-five.

He was a prodigy.

Again: I have known many chess and checker players who could sit with their eyes bandaged, or averted so they could see neither board nor pieces, and play a dozen or more games at the same time.

They are prodigies—not. They see in the air all the boards and pieces—not. They were born with the faculty of playing games blindfold—not.

What then is the explanation of their astonishing power? We call it visualizing, because that term may be made to mean what it suits us to make it mean. The players themselves say it is strictly a matter of *familiarity* with the game, plus practice. Persons who are so devoted to a game that they "feed" on it can not help being able to play blindfold, any more than a baseball fanatic can help having a picture in his

mind's eye of the diamond, the outfield, the players, and the ball in motion, when he reads the details of a game in his favorite newspaper.

As to mathematical prodigies: In the issue of *McClure's Magazine* for September, 1912, H. Addington Bruce tells what he has learned about "Lightning Calculators." We find that the principal celebrities of whom there are authentic records are:

Miguel Alberto Mantilla
William James Sidis
Jacques Inaudi
Arthur F. Griffith
Zerah Colburn
Henri Mondeux
Vito Mangiameli
Zacharias Dase
Jedediah Buxton
Truman Safford
George Bidder
André Ampère
Karl Gauss
Grandmange
Luigi Pierini

Following is a highly interesting extract from that article:

"The German calculator, Zacharias Dase, began giving public exhibitions at the age of fifteen. His ability to retain and mentally manipulate vast masses of figures has never been equaled by any other calculator on record. Most calculators, for example, have found it impossible to multiply in their heads two numbers containing more than ten or fifteen figures apiece. Excepting Dase, only one—that eighteenth-century marvel, Jedediah Buxton—has thus multiplied as high as two forty-figure numbers. Dase, however, went far beyond this, for at least once he mentally multiplied two one-hundred-figure numbers. Besides which he once extracted, entirely by mental processes, the square root of a sixty-figure number, and another time that of a hundred-figure number. The last operation took him only fifty-two minutes. Almost always, in fact, no matter how large the sums with which he had to deal, his calculations were performed in less time than would be required by the average mathematical expert working on paper. Oddly enough, outside of calculation, Dase was a sad igno-

ramus. Still, this has to be said of most lightning calculators. Of the more celebrated only five, so far as the records show, have displayed anything like high all-round mental ability—Truman Safford, George Bidder and his son of the same name, André Ampère, and Karl Gauss."

It is clear to me that these marvelous mental feats are not what many contented individuals choose to think they are—they are not magic. They are true processes—distinct operations—actual work. They are not instantaneous. They require time, however rapidly the mental mechanism may act.

Another important conclusion may be here put down in the words of the writer of the article:

"Yet—and most significant—in the case neither of the one-sided nor of the many-sided calculators does the constant strain they put on their minds seem to have been in the slightest injurious. Not one broke down mentally. Not one, so far as I have been able to ascertain, died before thirty, and only four—Griffith, Colburn, Dase, Mondeux—before forty. Ampère lived to sixty-one, Safford to sixty-five, Buxton to seventy, the

elder Bidder to seventy-two, and Gauss to seventy-eight. The average age, too, at which their power for rapid calculation was first observed was six—which means, of course, that their minds must have been more or less occupied with problems in calculation long before that time, or at the age probably of four or five. Gauss and Safford we know to have been calculating as early as three."

What force made these boys prodigies?

Why are there no girls in their class?

Who has not heard of "calculating boys?"

Who knows of any "calculating girls?"

Our writer does not bring up these questions or suggest them, but he does help us to a conclusion. He explains much when he says these children developed their powers along their definite line "solely because of an intense interest." In another place he calls it a "colossal" interest.

I like that explanation. I believe it is largely the true one. It helps me to understand why girls do not become mathematical prodigies—they are not interested in that line of achievement.

As to whether other boys, working in the same groove under the same conditions, could

have become lightning calculators—that may be debatable. Our authority, however, has the floor, and this is what he says:

"My own belief, to be specific, is that the mental processes of lightning calculators like little Miguel Mantilla differ not at all from those of ordinary human beings, that the only difference is an unusual facility of access to resources shared by everybody of normal mentality; and that this facility of access, in turn, depends on a factor utilizable by all."

Again:

"Now what is the explanation of such astounding mental mastery of the calendar (telling in less than a quarter of a minute the day of the week on which any date in any year falls), especially in one so young? Is it necessary to assume that Miguel Alberto Mantilla is the happy possessor of a supernormal faculty denied to the vast majority of men? Is it that his peculiar ability is perfectly normal, but the result of an exceptional inheritance? Or is it merely that he utilizes a power common to all mankind but not commonly drawn upon? And, in this case, would it be possible for

others, by appropriate training, to develop the same ‘gift,’ or one analogous to it? For myself, after a somewhat prolonged study of the whole problem of ‘lightning calculation,’ I am strongly inclined to answer both of these last two questions in the affirmative.”

It is always the same story—human beings become marvelous chess players, calculators, linguists, orators, poets, because they like to do what they do, and because they have become bent in that direction. Why they like to do this or that is because they can do it well. We all love to exercise, to develop, to use, the power we feel within us—but we are all nearer equal in mental endowment than we think we are.

As to physical accomplishments—fat men, lean men, big men, little men, tall men, short men, may all become graceful in the waltz—if they like it enough.

To my mind there are many “gifts” quite as staggering as those of the mathematical wizards. For example: I do not comprehend how a person can play the violin or paint a picture. I do not comprehend how an actor can learn his part; I do not comprehend how a schoolmaster can keep order

by merely looking fierce; and it is beyond the reach of my intellect to know how an ordinary box-factory Jen can with positive fidelity repeat every word of a prolonged controversy if she can get Liz to listen to her.

To me these persons and many others are as legitimately "freaks of nature" as the boy wonders who have been obsessed from infancy by a mania for counting and calculating. I have no interest in those matters. If I had a passion for them I believe I could learn to do them well enough to pass muster—but I also believe that I could not successfully compete with those who were better equipped at the start than myself.

I am unable to decide for myself at this time which came first—the chicken or the egg—the colossal interest, or the ability to do what a prodigy does with delight. Still, I stand pat on the old definition of genius—an infinite capacity for taking pains, or a capacity for taking infinite pains. It is all a matter of *attention*, any way.

PART V

CHILDREN'S DEPARTMENT

If you know the multiplication table up to ten, and can add correctly, say, 2 and 7 and 9, you do not have to make work of multiplying one long row of figures by another. It's fun, child's play, like tit-tat-to, or criss-cross, as we boys used to call it. I got the idea from the old Romans, about whom I shall tell you something a little later.

There is no need of a rule, as the same thing is done in all cases as in the following example, which shows how to multiply 74 by 8.

$$\begin{array}{r} 8 \times 7 \ 4 \\ \hline 5 \ 6 \ 2 \\ 3 \end{array}$$

This example shows that 8×7 is taken care of first, then 8×4 . The first product, 56, stands directly under 8; the second pro-

duct, 32, starts with 3 under 6, the 2 being placed, for convenience, at the top of the next column.

Before telling you what to do next, I will show you that larger examples are written down in the same way. Take, for instance, 78×74 , thus:

$$\begin{array}{r} 78 \times 74 \\ \hline 4562 \\ 93 \\ 28 \end{array}$$

Here, you see, you have started with 8×7 , as before, and have written down 56 and 32, as before. Then you take 7×7 , with 4 under the multiplier 7, and 9 in the next column. Then 7×4 , with 2 under 9, and 8 in the next column.

That is all there is to the writing down. You see you always start with the two figures separated by the sign \times . Putting that product down, you know just where to begin the next product without hesitation; and so on.

You multiply two figures together and write down two figures every time. You never have more than two figures to write down at one time, because even 9×9 makes only

two figures. If the product is only one figure, as, for instance, 3×2 , you write down two figures just the same, by making that product 06 instead of simply 6. This is for the purpose of guiding the eye without bothering the brain. So:

$$\begin{array}{r} 3 \times 2 \ 3 \\ \hline 0 \ 6 \ 9 \\ 0 \end{array}$$

Of course, it is not actually necessary to put down those ciphers, but the habit is a good one.

That brings us to the criss-cross part. In the first example you cross out 6 and 3 and put 9 under them. The answer is the figures not crossed out: 592.

The same thing is done in all cases. As soon as you see a chance to get rid of two or three figures making 10 or more, cross them out. If their sum is, say, 14, put the 4 under them and write the 1 in the next column to the left, or add it to any figures there and cross those out, putting their sum down as before. In this way you clean up as you go along, and do not have long columns of figures to add after you are all through multiplying.

Here is the second example completed:

$$\begin{array}{r} 78 \times 74 \\ \hline 4562 \\ 593 \\ 28 \\ \hline 77 \end{array}$$

As soon as I finished with 8 of the multiplier, I crossed it out, so I would not have to give it another thought at any time; likewise the 7. Then I crossed out 6 and 3 and 8, put 7 under them, and said, 1, 5, 9, 2 make 17; put down 7; 1 and 4 make 5.

That is all there is to the whole thing. Here is a large example complete:

$$\begin{array}{r} 95478 \times 748569 \\ \hline 63245624082 \\ 7435936447 \\ 358298855 \\ 321886523 \\ 145653468 \\ 6033883 \\ 7472046 \\ 4002239 \\ 22815 \\ 40505 \\ 75340 \\ 566 \\ 141 \\ 87 \\ 8 \end{array}$$

71471870982 Ans.

Now I am going to try to guess your thoughts. You are saying to yourself: "That's a queer-looking thing." Also: "Teacher doesn't multiply that way." Also: "Teacher wouldn't put down so many figures."

That is all true. I like to admit to children what I believe to be true. They like truth. Only grown-ups fear and hate it.

So there you are. But let me tell you something else. In shorthand writing the chief thing for many years was to make it as short as possible. After awhile they discovered that shorthand marks which are short to the eye may not be short for the hand. A dot, for example, which is a very short mark, is long in comparison with a dash, which is a long mark. Then it became known that long outlines are easiest, safest, fastest. Then we found out why—because they can be written without hesitation.

That long, queer-looking multiplication thing is as easy to do as rolling off a log. You may shorten it as much as you like, but what you gain for your eye in doing that you lose in ease of mind. There is no mental struggle going on in your brain as there is in your teacher's.

Another thing, you are not likely to make a mistake, because you do not have to carry anything in your mind while you are writing down the two figures, and you do not have columns to add after you get them all down. Furthermore, if you do not feel sure of the result, you do not try to trace the error, but do the example over again. The strain is on the fingers (which is cheap labor), rather than on the brain (which is real work).

I have just spoken of your teacher, and that reminds me that about forty years ago some one whose name I do not know invented three outlandish words, which you will find in the following sentence: When you see your teacher ask himer whether hiser way of doing multiplication is as safe and easy as hesh thinks this is. You see, I do not know whether your teacher is a man or a woman, and instead of "him or her," "his or her," "he or she," I use the three invented words. This really has nothing to do with our lecture, but I thought you would like to know about those little orphan words, although they are only neglected outcasts.

Heigh ho! So goes the world; is is is.

Well, now I will tell you something that

may surprise you. Of course, you believe what you teacher tells you, because hesh is good, and always does hiser best to tell you right. But did you ever wonder who told himer? (I won't use those little tramp words any more.)

You have learned what are called the Roman numerals, I, II, III, IV, V, VI, VII, VIII, IX, X, L, C, D, M. Your teacher tells you the old Romans could not readily calculate with their letters; that is, do addition and multiplication, for instance. She can prove it by showing you what arithmetics and encyclopedias say. If they did not perform these operations, it was not because they could not do them, but because they could make a fine calculating machine in two minutes with a handful of pebbles, coins, beans, or small pieces of anything. I shall speak of that machine later.

Whether the Romans ever wrote out tables for addition and multiplication, I do not know. That they knew what is shown in the tables I have made up and here present, I can not doubt.

ADDITION TABLE

And	I	II	III	IV	V	VI	VII	VIII	IX	X	L	C	D	M
i	ii	iii	iv	v	vi	vii	viii	ix	x	xi	ii	ci	di	mi
ii	iii	iv	v	vi	vii	viii	ix	x	xi	xii	iiii	clii	diis	miis
iii	iv	v	vi	vii	viii	ix	x	xi	xii	xiii	iiiiii	cliiii	diisii	miisii
iv	v	vi	vii	viii	ix	x	xi	xii	xiii	xiiii	iiiiiiii	cliiiiii	diisiiii	miisiiii
v	vi	vii	viii	viii	ix	x	xi	xii	xiii	xiiii	iiiiiiiiii	cliiiiiiii	diisiiiiii	miisiiiiii
vi	vii	viii	viii	viii	ix	x	xi	xii	xiii	xiiii	iiiiiiiiiiii	cliiiiiiiiii	diisiiiiiiii	miisiiiiiiii
vii	viii	viii	viii	viii	ix	x	xi	xii	xiii	xiiii	iiiiiiiiiiiiii	cliiiiiiiiiiii	diisiiiiiiiiii	miisiiiiiiiiii
viii	viii	viii	viii	viii	ix	x	xi	xii	xiii	xiiii	iiiiiiiiiiiiiiii	cliiiiiiiiiiiiii	diisiiiiiiiiiiii	miisiiiiiiiiiiii
ix	x	xii	xiii	xiii	xiv	xv	xvi	xvii	xviii	xviiii	iiiiiiiiiiiiiiiiii	cliiiiiiiiiiiiiiii	diisiiiiiiiiiiiiii	miisiiiiiiiiiiiiii
x	xii	xiii	xiii	xiv	xv	xvi	xvii	xviii	xviiii	xviiii	iiiiiiiiiiiiiiiiiiii	cliiiiiiiiiiiiiiiiii	diisiiiiiiiiiiiiiiii	miisiiiiiiiiiiiiiiii
ii	iii	iii	iii	iv	iv	iv	iv	iv	iv	iv	iiiiiiiiiiiiiiiiiiiiii	cliiiiiiiiiiiiiiiiiiii	diisiiiiiiiiiiiiiiiiii	miisiiiiiiiiiiiiiiiiii
1											c	cl	dl	ml
c	cl	clii	clii	clv	ev	evi	evii	eviii	cix	cix	cl	cc	dc	mc
d	di	diis	diis	div	dv	dvii	dviii	dviiii	dix	dix	dl	dc	m	md
m	mi	miis	miis	miiv	mv	mvii	mviii	mviiii	mx	mx	ml	mc	md	mm

MULTIPLICATION TABLE

Times	I	II	III	IV	V	VI	VII	VIII	IX	X	L	C	D	M
I	1	II	III	IV	V	VI	VII	VIII	IX	X	c	d	m	
II	II	IV	VI	VII	VIII	VII	VIII	VII	VIII	X	1	c	d	m
III	IV	VII	VIII	V	X	XII	XIV	XV	XVI	XVII	c	cc	m	II
IV	VII	V	XI	XII	XIII	XIV	XV	XVI	XVII	XVIII	cl	ccc	md	III
V	V	XII	XIII	XIV	XV	XVI	XVII	XVIII	XVII	XVIII	cl	ccc	md	IV
VI	VII	XII	XIII	XIV	XV	XVI	XVII	XVIII	XVII	XVIII	cc	cd	dd	II
VII	VII	XIV	XV	XVI	XVII	XVIII	XVII	XVIII	XVII	XVIII	cd	dd	dd	V
VIII	VIII	XVII	XVIII	XVII	XVIII	XVII	XVIII	XVII	XVIII	XVII	1	cccl	dc	VII
IX	IX	XVII	XVIII	XVII	XVIII	XVII	XVIII	XVII	XVIII	XVII	cccl	dccl	dc	VIII
X	X	XVII	XVIII	XVII	XVIII	XVII	XVIII	XVII	XVIII	XVII	cccl	dccl	dc	VII
1	1	c	cl	cc	cccl	ccc	cccl	cccl	cccl	cccl	cd	cd	cd	VIII
c	c	cc	ccc	cd	dc	dc	dc	dc	dc	dc	cm	cm	cm	IX
d	d	m	md	II	III	III	III	III	III	IV	IV	IV	IV	X
m	m	II	III	IV	V	VI	VII	VIII	VII	VIII	1	1	1	m

You learned in Roman notation that VI means $V + I$, and that IV means $-I + V$, or, if you prefer, $V - I$. That shows how the letters are united, the signs + and — being understood, or written in, if desirable. A bar over any letter or number of letters means thousands, thus, V is 5, but \bar{V} is 5,000. A number may be expressed in various ways, as desired. Thus, 8 may be VIII, or IIX, or IVIV.

Here are three examples, showing in detail what takes place when you add in Roman notation:

EXAMPLE
PROOF

$$\begin{array}{r} V + III \\ V + II \\ \hline X + V = XV \end{array} \qquad \begin{array}{r} 5 + 3 \\ 5 + 2 \\ \hline 10 + 5 = 15 \end{array}$$

EXAMPLE
PROOF

$$\begin{array}{r} X - I + X \\ X - I + V \\ \hline XX - II + XV = XXXIII \end{array} \qquad \begin{array}{r} 10 - 1 + 10 \\ 10 - 1 + 5 \\ \hline 20 - 2 + 15 = 33 \end{array}$$

$$\begin{array}{r} L + VI \\ LXXX + V \\ \hline D + L + I \\ \hline D + LL + LXXX + XII = DCXCII \end{array}$$

PROOF

$$\begin{array}{r} 50 + 6 \\ 80 + 5 \\ \hline .500 + 50 + 1 \\ \hline 500 + 100 + 80 + 12 = 692 \end{array}$$

If you had been taught to add letters, and knew nothing about figures, you would be able to add columns of letters easily and rapidly. Thus, you would see at a glance that $MDL + CL = MDCC$; that is, $1,550 + 150 = 1,700$.

As to multiplication in Roman notation, it is like the process in algebra. Now, in algebra (which is arithmetic with letters as well as figures), there is a kink which everybody knows, nobody denies, everybody can prove, nobody can explain—at least, so any one can understand the why of it. For example: — 3 times — 2 = + 6.

Let us try a little hot air.

Electricity is electricity, whether it is positive electricity, labeled +, or negative electricity labeled —. Number is number, whether it is positive number, labeled +, or negative number, labeled —.

That is the way real mathematicians think of those marks. But you learned to think of

them as meaning add and subtract, just as the other marks, \times and \div , mean multiply and divide. Look at this:

$$3 - 2$$

That means, you say, subtract 2 from 3. Then will you please tell me the meaning of this:

$$\begin{array}{r} 3 \\ - 2 \\ \hline \end{array}$$

Oh, you say, that's different. True, but the signs are the same, and you do not know whether to add, subtract, or multiply. Your trouble began the moment you learned that + means add and — means subtract.

That is not what they mean. If you insist that that is what they do mean, then you are obliged to admit that they also mean something else. But it is that something else which is their true meaning. Two meanings for the same mathematical sign is confusing, bad.

What the signs + and — show is the nature of the numbers you are dealing with. They indicate character. They answer the question: Is this a positive or a negative number? That is, above or below zero in the number ther-

momenter of your mind. So they are known to mathematicians as *the signs*. By the same token, some persons use — to indicate B.C. years and + to indicate A.D. years.

What $3 + 2$ really means is that + 3 and + 2 are to be united, not because of the sign +, but because they are brought together that way in a line. What $3 - 2$ really means is that + 3 and — 2 are to be united. If you write 32, you mean that 30 and 2 are united. A number without any sign is, of course, a positive number.

When you unite numbers, what you do is to find their *sum*. Note that word. Thus, add:

$$\begin{array}{r} 3 \\ - 2 \\ \hline \end{array}$$

The answer is 1, because — 2 (below zero) and + 3 (above zero), unite in your mental thermometer at zero, leaving 1 as their sum.

Add:

$$\begin{array}{r} - 3 \\ - 2 \\ \hline \end{array}$$

The answer is — 5, because 2 below unites with 3 below to make 5 below.

When you subtract one number from another, what you do is to find their *difference*. Note that word. Thus, subtract:

$$\begin{array}{r} 3 \\ - 2 \\ \hline \end{array}$$

The answer is $+ 5$, because $- 2$ (below) is separated from $+ 3$ (above), by a difference of 5.

Subtract:

$$\begin{array}{r} - 3 \\ - 2 \\ \hline \end{array}$$

The answer is $- 5$, because 2 above is separated from 3 below by 5, and it is $- 5$ because the nature of the ruling number ($- 3$) is negative. This may be not quite clear, but I think it is clear that the difference between $+ 2$ and $- 3$ is more negative than positive.

Subtract:

$$\begin{array}{r} - 3 \\ - 2 \\ \hline \end{array}$$

The difference between 2 below and 3 below is 1 below.

Of course you do not have to think of all that. A simple rule or two in algebra shows you just what to do with the signs in every kind of case.

Before dropping the subject, however, just a word more. The negative signs in multiplication act on each other in the same way that two negatives in language act. In each case two negatives make a positive. If you say, "I don't need no money," we know you do need money—that's positive. In -2 times -3 you have $+6$, because you negative a negative. That is, you make a negative not negative.

Having our multiplication table and our general principles to refer to, we shall have no difficulty in multiplying in Roman notation. It is well to begin at the left hand in order to get the highest numbers down first. So:

EXAMPLE

		PROOF
V	+	III
V	+	II
XXV	+	XV
X	+	VI
XXXV	+	XXI
= LVI		$35 + 21 = 56$

EXAMPLE

$$\begin{array}{r} X + \quad \text{IX} \\ X + \quad \text{IV} \\ \hline C + \quad \text{XC} \\ \text{XL} + \text{XXXVI} \\ \hline C + \text{CXXX} + \text{XXXVI} = \text{CCLXVI} \end{array}$$

PROOF

$$\begin{array}{r} 10 + \quad 9 \\ 10 + \quad 4 \\ \hline 100 + \quad 90 \\ \quad \quad \quad 40 + 36 \\ \hline 100 + 130 + 36 = 266 \end{array}$$

Let us multiply the year 1913 by the year 1492, without trying to make the work as short as we could make it if we were familiar with the process:

$$\begin{array}{r} M - C + M + X + \text{III} \\ M - C + D - X + \quad C + \text{II} \\ \hline \end{array}$$

It is apparent that the two M's in the top line may be at once united, and that the two C's (— C and + C) in the other line may be canceled. The work then is as follows:

EXAMPLE

$$\begin{array}{r}
 \text{MM} \quad \text{C} + \text{X} + \text{III} \\
 \text{M} \quad \text{D} + \text{X} + \text{II} \\
 \hline
 \text{MM} \quad \text{C} + \text{X} + \text{III} \\
 \text{M} \quad \text{L} + \text{V} + \text{MD} \\
 \hline
 \text{XX} + \text{M} - \text{C} = \text{XXX} \\
 \text{IV} - \text{CC} + \text{XX} + \\
 \hline
 \text{MM} = \text{CLXX} + \text{XX} + \text{IVCC} - \text{X} +
 \end{array}$$

This may now be twice simplified, to shc the full process, as follows:

$$\begin{array}{r}
 \text{MM} + \text{DCCCXXX} + \text{XXIVCC} - \text{IV} \\
 \text{MM DCCCLIV CXCVI} \textit{Ans.}
 \end{array}$$

PROOF

$$\begin{array}{r}
 \begin{array}{r}
 2,000 - 100 + 10 + 3 \\
 1,000 + 500 - 10 + 2 \\
 \hline
 \end{array} \\
 \begin{array}{r}
 2,000,000 - 100,000 + 10,000 + 3,000 \\
 1,000,000 - 50,000 + 5,000 + 1,500 \\
 - 20,000 + 1,000 - 100 - 30 \\
 \hline
 4,000 - 200 + 20 + 6 \\
 \hline
 \end{array} \\
 \begin{array}{r}
 3,000,000 + 170,000 + 20,000 + 4,200 - 10 + 6 \\
 2,830,000 + 24,200 - 4 \\
 \hline
 2,854,196 \textit{Ans.}
 \end{array}
 \end{array}$$

So I told my wife I wanted to buy a mathematical instrument used by the old Romans known as the abacus. Being a wise lady and

an excellent shopper, she told me that the proper place to get a mathematical instrument is a mathematical store. I wondered that I had not thought of that, but realized that I am not a competent shopper. I had no difficulty, however, in finding the mathematical store. Here was a 5 and 10 cent store, there a 5, 10, 15 and 25 cent store, on the next block a 3, 9, and 19 cent store, with a 1, 3 and 9 cent store close by, a nothing over 17 cent store a little farther on, and a come 7 come 11 cent store in the distance. I felt that my mission was an important one, and that I was probably a person of some consequence. Pride goeth before a fall. Without hesitation I walked into the first of the mathematical stores, and in a clear, manly voice, asked Mildred for an abacus. She continued to scratch her head with her lead pencil, as usual, and at the same time screeched:

“Ethel, have we got any abacusses?”

Several girls with strange looking hair at once came up and giggled at me. They did not seem to understand what I was there for. So I held up one of my hands and asked them whether they could imagine my fingers were wires. They said they could, easily. Then I

asked them to imagine there were ten rings on each finger.

The effect of that was dreadful. They exploded with hysterical mirth. I heard afterward that they said I was as good as the movies.

"Oh," I murmured, "I did not intend to be funny."

"That's why you are," said little, innocent, round-faced Bertha.

"Why!" she cried, "I know what the man wants. It's a *counting board*!"

I was allowed to buy a counting board and to pass out and on to the store having the highest number, 25, on its red sign. Here I asked for a counting board, in the hope of finding an abacus of the desired size, and was told that they did not carry such an article. Not wishing to take the paper off the one I had just obtained, I rattled it, at the same time explaining that the noise was caused by beads on wires. Then haughty Hortense scornfully told me that what I wanted was a *numeral frame*.

Educators seem to regard the abacus as of no account at the present day except as a toy. The fact is the abacus is the universal com-

puting machine of the human race from the earliest times. It is a perfect instrument. I do not doubt that if children were taught to add, subtract, multiply and divide on the abacus, they would prefer it to pencil and paper for fundamental operations.

The simplest form of abacus has ten beads or buttons on the top wire, representing units; ten on the next, representing tens, and so on. To multiply 29 by 6, for example, you say $6 \times 9 = 54$, putting up 4 buttons on the units wire and 5 on the tens wire. Then $6 \times 20 = 120$, put up 2 more on the tens wire and 1 on the hundreds wire. The answer, 174, is thus registered before you.

By modifications of one kind or another, the abacus can be used for complex operations. The Romans sometimes used pebbles or other small objects on a board in preference to buttons on a string. They divided some of their computing boards to show a separate section of 5's, and had another section for ounces and what we call business fractions.

Most writers on mathematics give the Romans small credit as calculators; but, if called upon to guess, I should say that the Romans

knew how to reckon better than we think they did.

The word calculus was used by the Romans for the pebbles or pieces of stone of one of their abacus tables. That word with us means any branch of mathematics, but especially that powerful device known from the days of Newton and Leibnitz as the calculus.

Now the calculus, or simply calculus, does not belong to the class called "higher mathematics." Calculus is advanced practical mathematics, while much of the "higher mathematics" is little short of mere metaphysics; the chief value of which is that it furnishes certain persons the desired requisites for their imaginary mental aristocracy clubs.

There are two kinds of objects—those outside the mind and those inside the mind. Those outside the mind are called physical objects—that is, they have some kind of a body. Those inside the mind are called mental objects—that is, they have no actual body. Gold, water, apples, and even shadows, are physical objects. Time, quantity, number, and even space, are mental objects. We can not know what they actually are, but we can know a good deal about how they behave.

In arithmetic we see number on the march. It takes long steps, 1, 2, 3, etc., or short steps, fractions; but long or short, they are steps.

In the calculus we see number moving along like a river, flowing, not stepping. That view of number has a soothing effect on the imagination, for it relieves our distress when we wonder how anything could be so small that it could not be divided, or so far away that it could not be farther.

By looking at number as a continuous thing, as flowing instead of stepping along, the calculus gets an almost magical power.

Here is the simplest kind of a problem I have been able to invent.

A boy receives ten pennies from his mother. His father tells him to separate the ten pennies into two piles, and multiply the number in one pile by the number in the other pile. The product is to be the number of pennies his father will give him.

How many pennies must the boy put in each pile to get the largest possible amount from his father.

You can quickly find the answer by inspection, by experiment, or by logic (geometry), because 10 is a small number. Do you know

of any rule in arithmetic that covers this kind of problem? I gave it to some young persons well trained in elementary algebra, and they said it did not have enough "terms." Assuming, for the purpose of illustration, that this childish problem is beyond the reach of common arithmetic and algebra, we are thus enabled to get a glimpse of the peculiar power of the calculus.

Number flows. Hence you are able to detach portions of it so small that they behave in a way similar to that of the supposed atoms of matter. You know that an atom of matter can not exist alone, but must be in a combination of some kind. The lone atom vanishes. It violates common sense to say that something becomes nothing by merely saying so. Now we are on treacherous ground. That is the spot where the deadlocks in philosophy occur. It is so, if it isn't so, just because you say so. Any way, the isolated atom of number is dropped from the calculation—disappears, like a dead microbe.

Calculus is the chemistry of number. In order to state our little problem mathematically, we say:

$$X = \text{one pile}$$
$$10 - X = \text{the other pile}$$

Multiplying one pile by the other, we get the amount desired, thus:

$$10X - X^2$$

Now instead of $10X$, we may attach anything, say a * to the x to show that it is an x atom magnified 10 times. In x^2 we have a group of atoms, and right there we find the secret of x all wrapped up, so to speak. In this case the atom, instead of being magnified by 10, is magnified by two x 's. So we put that atom down as $2x^*x$. Now we can make the universal tool of mathematicians—the equation.

Atoms are equal to atoms, so

$$10^*X - 2X^*X = \text{what } *X$$

It is all over now. Merely brush away the microbes; that is, cancel out the $*x$'s, and you have:

$$\begin{aligned} 10 - 2X &= 0 \\ - 2X &= - 10 \\ X &= 5 \end{aligned}$$

You are dismissed.

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